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Computation of transient viscoelastic flow problems approximated by a VMS stabilized Finite Element formulation using time-dependent subgrid-scales for monolithic and fractional step schemes

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Outline

What is a viscoelastic fluid?

- Viscoelastic fluids are a specific type of **non-Newtonian fluids** that exhibit a combination of elastic and viscous effects.
  - **Visco**: friction, irreversibility, loss of memory.
  - **Elastic**: recoil, internal energy storage.

- This combination of properties is explained by a **complex internal structure**.
- They have **memory**. The state-of-stress depends on the flow history.

wikipedia

Modelling of polymeric fluid flows

- Momentum equation:

$$\rho \frac{\partial \boldsymbol{u}}{\partial t} + \rho \boldsymbol{u} \cdot \nabla \boldsymbol{u} - \nabla \cdot \mathbf{T} + \nabla p = \boldsymbol{f}$$

Deviatoric extra stress tensor

$$\mathbf{T} = 2\eta(\nabla^s \boldsymbol{u})$$

Newtonian viscous fluids

- Continuity equation:

$$\nabla \cdot \boldsymbol{u} = 0$$

$$\mathbf{T} = 2\eta_s(\nabla^s \boldsymbol{u}) + \boldsymbol{\sigma}$$

Polymeric fluids

- Constitutive equation:

$$\frac{1}{2\eta_p}(1 + \mathfrak{h}(\boldsymbol{\sigma})) \cdot \boldsymbol{\sigma} - \nabla^s \boldsymbol{u} + \frac{\lambda}{2\eta_p}\left(\frac{\partial \boldsymbol{\sigma}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{\sigma} - \boldsymbol{\sigma} \cdot \nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^{\mathrm{T}} \cdot \boldsymbol{\sigma}\right) = \mathbf{0}$$

$$\mathfrak{h}(\boldsymbol{\sigma}) = 0$$

Oldroyd-B

$$\mathfrak{h}(\boldsymbol{\sigma}) = \frac{\varepsilon\lambda}{\eta_p}\boldsymbol{\sigma}$$

Giesekus

$$\mathfrak{h}(\boldsymbol{\sigma}) = \frac{\varepsilon\lambda}{\eta_p}\mathrm{tr}(\boldsymbol{\sigma})$$

Phan-Thien-Tanner

The Weissenberg number and HWNP

- We is small: viscous effect relevant, elastic behavior small.
- If $\mathrm{We} > 1$: problems become extremely complicated.

Weissenberg number

$$\mathrm{We} = \frac{\lambda U}{L}$$

Problem: The High Weissenberg Number Problem (HWNP)

"Solution": the Log-Conformation Reformulation (LCR)

Motivation and goal

Classical residual-based stabilized methods may experience **difficulties when the time step is small** relative to the spatial grid size.

- Bochev et al. demonstrate that $\delta t > Ch^2$ is **a sufficient condition** to avoid instabilities.
- For anisotropic space-time discretizations, this inequality **is not necessarily satisfied**.

Goal

1. **Design of new stabilization techniques**.
2. Design **fractional step schemes** in order to reduce the expensive computational cost.

Spatial and temporal discretizations

Spatial discretization

Galerkin finite element approximation. It consists in finding $\boldsymbol{U}_h : (0, T) \longrightarrow \boldsymbol{\mathcal{X}}_h$,

$$\underbrace{(\mathcal{G}(\boldsymbol{U}_h), \boldsymbol{V}_h)}_{\text{Temporal terms}} + \underbrace{B(\boldsymbol{u}_h; \boldsymbol{U}_h, \boldsymbol{V}_h)}_{\text{Semi-linear form}} = L(\boldsymbol{V}_h),$$

for all $\boldsymbol{V}_h \in \boldsymbol{\mathcal{X}}_h$.

Time discretization

Monolithic and **fractional step** time discretization. BDF1 and BDF2 schemes have been employed in this work.

Stabilization technique: Variational Multi-Scale (VMS) Methods

- Objective: to approximate the components of the continuous problem solution that cannot be resolved by the finite element mesh.
- Unknown splitting: $\boldsymbol{U} = \underbrace{\boldsymbol{U}_h}_{\in \boldsymbol{\mathcal{X}_h}} + \underbrace{\tilde{\boldsymbol{U}}}_{\in \tilde{\boldsymbol{\mathcal{X}}}}$ and $\boldsymbol{\mathcal{X}} = \boldsymbol{\mathcal{X}_h} \bigoplus \tilde{\boldsymbol{\mathcal{X}}}$.

$$\underbrace{(\mathcal{G}(\boldsymbol{U}_h), \boldsymbol{V}_h) + B(\boldsymbol{u}_h; \boldsymbol{U}_h, \boldsymbol{V}_h)}_{\text{Galerkin terms}} + \underbrace{\langle \mathcal{G}(\tilde{\boldsymbol{U}}), \boldsymbol{V}_h \rangle + \sum_K \langle \tilde{\boldsymbol{U}}, \underbrace{\mathcal{L}^*(\boldsymbol{u}_h; \boldsymbol{V}_h)}_{\text{adjoint operator of } \mathcal{L}} \rangle_K}_{\text{Stabilization terms}} = L(\boldsymbol{V}_h)$$

$$\boxed{\frac{\partial \tilde{\boldsymbol{U}}}{\partial t} + \boldsymbol{\alpha}^{-1} \tilde{\boldsymbol{U}} = \tilde{P}[\mathbf{F} - \mathcal{G}(\boldsymbol{U}_h) - \mathcal{L}(\boldsymbol{u}_h; \boldsymbol{U}_h)]}$$

Sub-grid scale

- $\tilde{P}$ is the $L^2 -$ projection onto the space of sub-grid scales,
- $\boldsymbol{\alpha}$ is a matrix computed within each element,
- $\mathcal{L}$ is the operator associated with the problem.

Dynamic subscales for residual-based stabilized formulation

$$\overbrace{\mathcal{G}(\boldsymbol{U}_h, \boldsymbol{V}_h) + B(\boldsymbol{u}_h; \boldsymbol{U}_h, \boldsymbol{V}_h)}^{\text{Galerkin terms}} + \overbrace{(\mathcal{G}(\tilde{\boldsymbol{U}}), \boldsymbol{V}_h) + S_1(\boldsymbol{u}_h; \boldsymbol{U}_h, \boldsymbol{V}_h) + S_2(\boldsymbol{U}_h, \boldsymbol{V}_h) + S_3(\boldsymbol{u}_h; \boldsymbol{U}_h, \boldsymbol{V}_h)}^{\textbf{Stabilization terms}} = \underbrace{L(\boldsymbol{V}_h)}_{\text{Gal. term}}$$

$$S_1(\hat{\boldsymbol{u}}_h; \boldsymbol{U}_h, \boldsymbol{V}_h) = \sum_K \left\langle \tilde{\boldsymbol{u}}, -\nabla \cdot \chi_h + 2\beta\eta_0 \nabla \cdot (\nabla^s \boldsymbol{v}_h) + \rho \hat{\boldsymbol{u}}_h \cdot \nabla \boldsymbol{v}_h + \nabla q_h \right\rangle_K$$

$$\rho \frac{\partial \tilde{\boldsymbol{u}}}{\partial t} + \alpha_1^{-1} \tilde{\boldsymbol{u}} = \tilde{P}\Big(\overbrace{\boldsymbol{F}_1 - \rho \frac{\partial \boldsymbol{u}_h}{\partial t} - \mathcal{L}_1(\boldsymbol{u}_h; \boldsymbol{U}_h)}^{\text{Momentum eq. residual}}\Big),$$

$$\tilde{\boldsymbol{u}}^{n+1} = \underbrace{\left(\rho \frac{1}{\delta t} + \frac{1}{\alpha_1^{n+1}}\right)^{-1}}_{\alpha_{1\text{dyn}}} \left(\rho \frac{1}{\delta t} \tilde{\boldsymbol{u}}^n - \rho \tilde{P}\left(\boldsymbol{F}_1 - \rho \frac{\partial \boldsymbol{u}_h}{\partial t} - \mathcal{L}_1(\boldsymbol{u}_h; \boldsymbol{U}_h)\right)\right)$$

Discretization using a BDF1 scheme

Stabilized formulation: Residual-based vs Split OSS

$$\overbrace{(\mathcal{G}(\boldsymbol{U}_h), \boldsymbol{V}_h) + B(\boldsymbol{u}_h; \boldsymbol{U}_h, \boldsymbol{V}_h)}^{\text{Galerkin terms}} + \overbrace{(\mathcal{G}(\tilde{\boldsymbol{U}}), \boldsymbol{V}_h) + S_1(\boldsymbol{u}_h; \boldsymbol{U}_h, \boldsymbol{V}_h) + S_2(\boldsymbol{U}_h, \boldsymbol{V}_h) + S_3(\boldsymbol{u}_h; \boldsymbol{U}_h, \boldsymbol{V}_h)}^{\textbf{Stabilization terms}} = \underbrace{L(\boldsymbol{V}_h)}_{\text{Gal. term}}$$

$$S_1(\hat{\boldsymbol{u}}_h; \boldsymbol{U}_h, \boldsymbol{V}_h) = \sum_K \alpha_{1\text{dyn}} \Big\langle \rho \frac{1}{\delta t} \tilde{\boldsymbol{u}}^n + \tilde{P}\Big[\rho \frac{\partial \boldsymbol{u}_h}{\partial t} \overbrace{- \frac{\eta_p}{\lambda_0} \nabla \cdot (\exp(\boldsymbol{\psi}_h))}^{\tilde{\boldsymbol{u}}_1} - 2\eta_s \nabla \cdot (\nabla^s \boldsymbol{u}_h) \overbrace{+ \rho \hat{\boldsymbol{u}}_h \cdot \nabla \boldsymbol{u}_h}^{\tilde{\boldsymbol{u}}_2},$$

$$\overbrace{+ \nabla p_h}^{\tilde{\boldsymbol{u}}_3}\Big] - \nabla \cdot \boldsymbol{\chi}_h + 2\eta_s \nabla \cdot (\nabla^s \boldsymbol{v}_h) + \rho \hat{\boldsymbol{u}}_h \cdot \nabla \boldsymbol{v}_h + \nabla q_h \Big\rangle_K$$

1. The **residual-based stabilization** contemplates all terms.
2. **Split OSS stabilization**: neglect the cross local inner-product terms as well as some other terms that do not contribute to stability. In that case $\tilde{P} = P_h^{\perp}$.

Fractional step. Algebraic system

Momentum $$M_{\boldsymbol{u}} \frac{\delta_k}{\delta t} \mathbf{U}^{n+1} + K_{\boldsymbol{u}} \left(\mathbf{U}^{n+1}\right) \mathbf{U}^{n+1} + G\mathrm{P}^{\mathrm{n+1}} - D_{\boldsymbol{\psi}}^{\mathrm{E}} \boldsymbol{\Psi}^{\mathrm{n+1}} = \mathbf{F}_{\boldsymbol{u}}^{\mathrm{E}}, \quad (1)$$

Continuity $$D\mathbf{U}^{n+1} = \mathbf{0}, \quad (2)$$

Constitutive $$M_{\psi}^{\mathrm{E}} \frac{\delta_k}{\delta t} \boldsymbol{\Psi}^{n+1} + K_{\psi}^{\mathrm{E}} \left(\mathbf{U}^{n+1}\right) \boldsymbol{\Psi}^{n+1} - S\mathbf{U}^{n+1} = \mathbf{F}_{\boldsymbol{\psi}}^{\mathrm{E}}. \quad (3)$$

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & 0 \\ A_{31} & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{U}^{n+1} \\ \boldsymbol{\Psi}^{n+1} \\ \mathrm{P}^{n+1} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_1^{n+1} \\ \mathbf{F}_2^{n+1} \\ \mathrm{F}_3^{\mathrm{n+1}} \end{bmatrix}$$

$$A_{11} = \frac{1}{\gamma_k \delta t} M_{\boldsymbol{u}} + K_{\boldsymbol{u}} \left(\mathbf{U}^{n+1}\right), \qquad A_{12} = -D_{\boldsymbol{\psi}}^{\mathrm{E}},$$

$$A_{21} = -S, \qquad A_{22} = \frac{1}{\gamma_k \delta_K} M_{\boldsymbol{\psi}}^{\mathrm{E}} + K_{\boldsymbol{\psi}}^{\mathrm{E}} (\mathbf{U})^{n+1},$$

$$A_{13} = G, \qquad A_{31} = D,$$

$$\mathbf{F}_1 = \mathbf{F}_{\boldsymbol{u}}^{\mathrm{E}} + \frac{1}{\delta t \gamma_k} \left(\sum_{i=0}^{k-1} \varphi_k^i \mathbf{U}^{n-i} \right), \qquad \mathrm{F}_3 = 0,$$

$$\mathbf{F}_2 = \mathbf{F}_{\boldsymbol{\psi}}^{\mathrm{E}} + \frac{1}{\delta t \gamma_k} \left(\sum_{i=0}^{k-1} \varphi_k^i \boldsymbol{\Psi}^{n-i} \right).$$

Fractional step. Equivalent form

$$M_{\boldsymbol{u}}\frac{\delta_k}{\delta t}\tilde{\mathrm{U}}^{n+1}+K_{\boldsymbol{u}}\left(\tilde{\mathrm{U}}^{n+1}\right)\tilde{\mathrm{U}}^{n+1}+G\hat{\mathrm{P}}^{n+1}_{k'-1}-D^{\mathrm{E}}_{\psi}\hat{\boldsymbol{\Psi}}^{n+1}_{k'-1}=\mathbf{F}^{\mathrm{E}}_{\boldsymbol{u}}, \quad (4)$$

$$M_{\boldsymbol{u}}\frac{\delta_k}{\delta t}\left(\mathbf{U}^{n+1}-\tilde{\mathrm{U}}^{n+1}\right)+N_{\boldsymbol{u}}+G\left(\mathrm{P}^{n+1}-\hat{\mathrm{P}}^{n+1}_{k'-1}\right)-D^{\mathrm{E}}_{\psi}\left(\boldsymbol{\Psi}^{n+1}-\hat{\boldsymbol{\Psi}}^{n+1}_{k'-1}\right)=\mathbf{0}, \quad (5)$$

$$M^{\mathrm{E}}_{\psi}\frac{\delta_k}{\delta t}\tilde{\boldsymbol{\Psi}}^{n+1}+K^{\mathrm{E}}_{\psi}\left(\tilde{\mathrm{U}}^{n+1}\right)\tilde{\boldsymbol{\Psi}}^{n+1}-S\tilde{\mathrm{U}}^{n+1}=\mathbf{F}^{\mathrm{E}}_{\psi}, \quad (6)$$

$$M^{\mathrm{E}}_{\psi}\frac{\delta_k}{\delta t}\left(\boldsymbol{\Psi}^{n+1}-\tilde{\boldsymbol{\Psi}}^{n+1}\right)+\mathbf{N}^{n+1}_{\psi}-S\left(\mathbf{U}^{n+1}-\tilde{\mathrm{U}}^{n+1}\right)=\mathbf{0}, \quad (7)$$

$$-D\tilde{\mathrm{U}}^{n+1}+\gamma_k\delta tDM^{-1}_{\boldsymbol{u}}\mathbf{N}^{n+1}_{\boldsymbol{u}}+\gamma_k\delta tDM^{-1}_{\boldsymbol{u}}G\left(P^{n+1}-\hat{\mathrm{P}}^{n+1}_{k'-1}\right)$$
$$-\gamma_k\delta tDM^{-1}_{\boldsymbol{u}}D^{\mathrm{E}}_{\psi}\left(\boldsymbol{\Psi}^{n+1}-\hat{\boldsymbol{\Psi}}^{n+1}_{k'-1}\right)=\mathbf{0}, \quad (8)$$

$\tilde{\mathrm{U}}^{n+1}$ and $\tilde{\boldsymbol{\Psi}}^{n+1}$ are the **auxiliary variables** that later must be corrected.
$\hat{g}^{n+1}_{k'-1}$ are the **extrapolated variables**, where the order of the extrapolation is $k'-1$ at time t^{n+1}.

(4) + (5) $\longrightarrow$ (1)Momentum
(6) + (7) $\longrightarrow$ (3)Constitutive
(8) + $\gamma_k\delta tD$ (5) $\longrightarrow$ (2)Continuity

Fractional step. Algorithm (I). Intermediate steps

1. **Compute the intermediate velocity** using extrapolated values of the pressure and ψ variable.

$$(4)\ M_{\boldsymbol{u}} \frac{\delta_k}{\delta t} \tilde{\mathrm{U}}^{n+1} + K_{\boldsymbol{u}} \left(\tilde{\mathrm{U}}^{n+1} \right) \tilde{\mathrm{U}}^{n+1} + G \hat{\mathrm{P}}^{n+1}_{k'-1} - D^{\mathrm{E}}_{\psi} \hat{\boldsymbol{\Psi}}^{n+1}_{k'-1} = \mathbf{F}^{\mathrm{E}}_{\boldsymbol{u}} \longrightarrow \tilde{\mathrm{U}}^{n+1}$$

2. **Compute the intermediate** ψ using the intermediate velocity computed in the previous step.

$$(6)\ M^{\mathrm{E}}_{\psi} \frac{\delta_k}{\delta t} \tilde{\boldsymbol{\Psi}}^{n+1} + K^{\mathrm{E}}_{\psi} \left(\tilde{\mathrm{U}}^{n+1} \right) \tilde{\boldsymbol{\Psi}}^{n+1} - S \tilde{\mathrm{U}}^{n+1} = \mathbf{F}^{\mathrm{E}}_{\psi} \longrightarrow \tilde{\boldsymbol{\Psi}}^{n+1}$$

3. **Compute the intermediate pressure** using both intermediate velocities and ψ computed in the two previous steps:

$$(8)\ - D \tilde{\mathrm{U}}^{n+1} + \gamma_k \delta t D M_{\boldsymbol{u}}^{-1} \mathbf{N}^{n+1}_{\boldsymbol{u}} + \gamma_k \delta t D M_{\boldsymbol{u}}^{-1} G \left(\tilde{P}^{n+1} - \hat{\mathrm{P}}^{n+1}_{k'-1} \right)$$
$$- \gamma_k \delta t D M_{\boldsymbol{u}}^{-1} D^{\mathrm{E}}_{\psi} \left(\tilde{\boldsymbol{\Psi}}^{n+1} - \hat{\boldsymbol{\Psi}}^{n+1}_{k'-1} \right) = \mathbf{0} \longrightarrow \tilde{\mathrm{P}}^{n+1}$$

Fractional step. Algorithm (II). Correction steps

4 **Compute the velocity correction**:

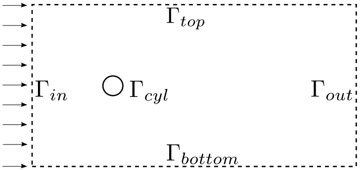
$$(5)\ M_{\boldsymbol{u}} \frac{\delta_k}{\delta t}\left(\mathbf{U}^{n+1} - \tilde{\mathrm{U}}^{n+1}\right) + \mathbf{N}_{\boldsymbol{u}}^{n+1} + G\left(\tilde{\mathrm{P}}^{n+1} - \hat{\mathrm{P}}_{k'-1}^{n+1}\right) - D_{\boldsymbol{\psi}}^{\mathrm{E}}\left(\tilde{\boldsymbol{\Psi}}^{n+1} - \hat{\boldsymbol{\Psi}}_{k'-1}^{n+1}\right) = \mathbf{0} \longrightarrow \mathbf{U}^{n+1}$$

5 **Compute the ψ correction**:

$$(7)\ M_{\boldsymbol{\psi}}^{\mathrm{E}} \frac{\delta_k}{\delta t}\left(\boldsymbol{\Psi}^{n+1} - \tilde{\boldsymbol{\Psi}}^{n+1}\right) + \mathbf{N}_{\boldsymbol{\psi}}^{n+1} - S\left(\mathbf{U}^{n+1} - \tilde{\mathrm{U}}^{n+1}\right) = \mathbf{0} \longrightarrow \boldsymbol{\Psi}^{n+1}$$

6 **Pressure correction**: $\mathrm{P}^{\mathrm{n+1}} = \tilde{\mathrm{P}}^{\mathrm{n+1}} \longrightarrow \mathrm{P}^{\mathrm{n+1}}$

Flow over a cylinder. Main features.



- The computational domain is a rectangle of length 16 and width 8, with a unitary cylinder centered vertically.
- The boundary conditions of the problem are:
  - The inflow velocity is $u_x = 1$ and $u_y = 0$.
  - The top and the bottom boundaries are considered fictitious walls.
  - For the outflow boundary the velocity is free in both components.
  - Non-slip conditions are set in the wall of the cylinder .
- The viscoelastic fluid parameters are: $\rho = 1$, $\beta = 0.5$ and $\eta_0 = 0.01$.
- Re=100, We $\in \{0.125, 0.165, 0.25, 0.5\}$.
- Spatial discretization: Coarse mesh. $h_{min} = 0.01$ and $h_{max} = 0.4$.
- Temporal discretization: $\delta t \in \{0.05, 0.025, 3.125 \times 10^{-3}, 1.562 \times 10^{-3}\}$

Flow over a cylinder. Monolithic scheme

Comparing stabilizations

P1 elements	Time step (δt)			
Method	0.050	0.0250	3.125×10^{-3}	1.562×10^{-3}
Static-OSS	Solved	Failed	-	-
Dyn-OSS	Solved	Solved	Solved	Solved
Static-SOSS	Solved	Solved	Solved	Failed
Dyn-SOSS	Solved	Solved	Solved	Solved

Table: Solved and failed cases $\mathrm{We} = 0.125$, $\alpha_{1,\mathrm{min}} \approx 1.156 \times 10^{-3}$.

Comparing formulations

	Weissenberg (We)			
Formulation	0.125	0.165	0.25	0.5
Std-Static	Solved	Failed	-	-
Std-Dyn	Solved	Solved	Solved	Failed
LCR-Static	Solved	Solved	Failed	-
LCR-Dyn	Solved	Solved	Solved	Solved

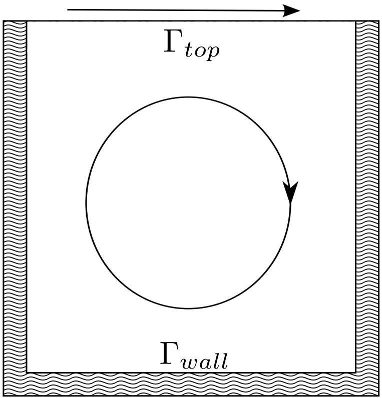
Table: Solved and failed cases for S-OSS formulations, dynamic and quasi-static, $\delta t = 0.1$.

Conclusions

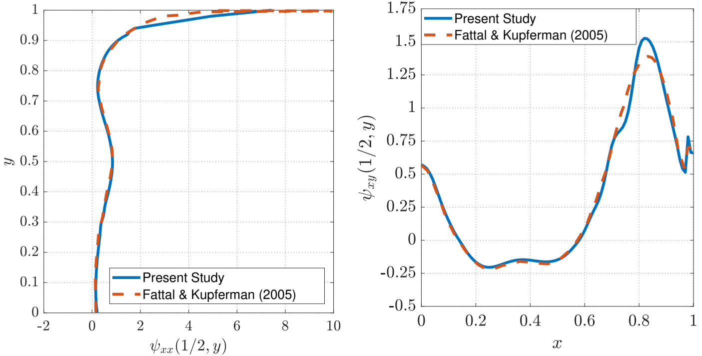
- The most unstable stabilization is the **quasi-static** + OSS stabilization.
- **Dynamic formulations** are more efficient avoiding elastic instabilities.

Lid-driven cavity flow problem. Main features

- The computational domain: fluid confined in the unit square.
- The boundary conditions are:
  - On the top boundary, velocity is prescribed in the x direction.
  - At the walls, velocity is set to zero in both components.
- $\mathrm{We} = 1.0$ and $\mathrm{Re} = 0$.
- Spatial discretization: a structured mesh composed of 10000 bilinear Q1 elements.
- Temporal discretization: $\delta t = 0.0025$



Lid-driven cavity flow problem. Case Re=0



	Stabilization S-OSS	
Formulation	Quasi-static	Dynamic
Standard	Failed - time step 265	Failed - time step 1316
Logarithmic	Failed - time step 340	Solved

Table: Comparison between different formulations, $\mathrm{We} = 1.0$, $\delta t = 0.0025$. The time step at which convergence fails is indicated.

Lid-driven cavity flow problem. Monolithic vs Fractional schemes

		Stabilization S-OSS	
Scheme	**Formulation**	Quasi-static	Dynamic
Monolithic	Standard	Failed - time step 265	Failed - time step 1316
Monolithic	LCR	Failed - time step 340	Solved
Fractional Step	Standard	Failed - time step 692	Failed - time step 1781
Fractional Step	LCR	Failed - time step 593	Solved

Table: Comparison between different formulations and schemes with $\mathrm{We} = 0.5$, Re $= 0$, $\delta t = 0.0025$ considering a mesh of 35×35. The time step at which convergence fails is indicated in each case.

Case	Total time ratio	Solver time ratio
We=0.5, Re=0.0	0.48	0.10
We=1.0, Re=100	0.49	0.07

Table: Ratio of CPU times. Computational mesh 100×100.

Conclusions

- **Dynamic sub-scales** allow solving problems where two different sources of instability can appear simultaneously.
- **Combination of LCR and dynamic subscales** in term-by-term stabilization is capable of solving problems with **higher elasticity than other options**.
- **Fractional step methods** for the LCR have been designed using a purely algebraic approach in order to **reduce the computational cost**.

Related publications

1. Moreno L., Codina R., Baiges J. & Castillo E. (2019). Logarithmic conformation reformulation in viscoelastic flow problems approximated by a VMS-type stabilized finite element formulation. *Computer Methods in Applied Mechanics and Engineering*, 354, 706-731.
2. Moreno L., Codina R. & Baiges J. (2020). Solution of transient viscoelastic flow problems approximated by a term-by-term VMS stabilized finite element formulation using time-dependent subgrid-scales. *Computer Methods in Applied Mechanics and Engineering*, 367, 113074.
3. Codina, R., & Moreno, L. Stability Analysis of time dependent linearized viscoelastic flow problems using a stabilized finite element formulation in space. *ESAIM. Mathematical Modelling and Numerical Analysis*, Submitted.
4. Castillo, E., Moreno, L., Baiges, J., & Codina, R. (2021). Stabilised Variational Multi-Scale Finite Element Formulations for Viscoelastic Fluids. *Archives of Computational Methods in Engineering*, 1-33.

References

- T.J.R. Hughes, G.R. Feijóo, L. Mazzei, and J.B. Quincy. The variational multiscale method: a paradigm for computational mechanics. *Computer Methods in Applied Mechanics and Engineering*, 166:3–24, 1998.
- Hulsen M. A. , Fattal R. & R. Kupferman. (2005) Flow of viscoelastic fluids past a cylinder at high weissenberg number: stabilized simulations using matrix logarithms. *Journal of Non-Newtonian Fluid Mechanics*, 127(1):27–39.
- R. Fattal and R. Kupferman. Constitutive laws for the matrix-logarithm of the conformation tensor. *Journal of Non-Newtonian Fluid Mechanics*, 123(2-3):281–285, 2004.
- Castillo, E., Codina, R. (2015). First, second and third order fractional step methods for the three-field viscoelastic flow problem. *Journal of Computational Physics*, 296. https://doi.org/10.1016/j.jcp.2015.04.027
- Codina, R., Principe, J., Guasch, O., Badia, S. (2007). Time dependent subscales in the stabilized finite element approximation of incompressible flow problems. *Computer Methods in Applied Mechanics and Engineering*, 196(21–24). https://doi.org/10.1016/j.cma.2007.01.002
- P. B. Bochev, M. D. Gunzburger, and J. N. Shadid. (2004) On inf–sup stabilized finite element methods for transient problems. *Computer Methods in Applied Mechanics and Engineering* 193.15-16: 1471-1489.

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Thank you for your attention!!

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